

(3 Hours)

[Total Marks: 80]

N.B (1) Question No. 1 is compulsory.

(2) Solve any **three** questions out of remaining **five** questions.

(3) Assumptions made should be clearly stated.

(4) Figures to the right indicate full marks.

Q.1 (a) Two dice are rolled, find the probability that the sum is

(i) Equal to 1 (ii) Equal to 4 (iii) Less than 13

[6M]

(b) Use the laws of logic to show that

 $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$ is a tautology

[6M]

(c) Determine the matrix of the partial order of divisibility on the set A. Draw the Hasse diagram of the Poset. Indicate those which are chains

[8M]

(1) $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ (2) $A = \{3, 6, 12, 36, 72\}$ Q.2 (a) Find the complement of each element in D_{42} .

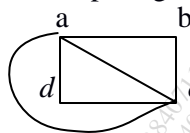
[6M]

(b) Let Q be the set of positive rational numbers which can be expressed in the form $2^a 3^b$, where a and b are integers. Prove that algebraic structure (Q, \cdot) is a group. Where \cdot is multiplication operation.

[6M]

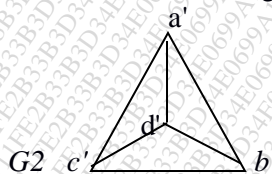
(c) Define isomorphic graphs. Show whether the following graphs are isomorphic or not.

[8M]



G1

Fig (a)

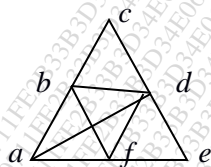


G2

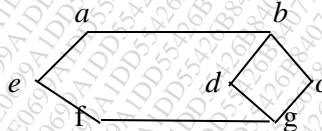
Fig (b)

Q.3 (a) Determine which of the following graph contains an Eulerian or Hamiltonian circuit.

[6M]



Fig(a)



Fig(b)

(b) For all sets A, X and Y show that

$$A \times (X \cap Y) = (A \times X) \cap (A \times Y)$$

[6M]

(c) Let $f(x) = x+2$, $g(x) = x-2$ and $h(x) = 3x$ for $x \in \mathbb{R}$, Where \mathbb{R} = Set of real numbers. Find

[8M]

 $(g \circ f), (f \circ g), (f \circ f), (g \circ g), (f \circ h), (h \circ g), (h \circ f), (f \circ h \circ g)$ Q.4 (a) Let R is a binary relation. Let $S = \{(a, b) \mid (a, c) \in R \text{ and } (c, b) \in R \text{ for some } c\}$ Show that if R is an equivalence relation then S is also an equivalence relation.

[6M]

[TURN OVER]

- (b) Determine the generating function of the numeric function a_r , where [6M]

(i) $a_r = 3^r + 4^{r+1}$, $r \geq 0$

(ii) $a_r = 5$, $r \geq 0$

- (c) Consider the (3, 6) encoding function $e: B^3 \rightarrow B^6$ defined by [8M]

$e(000) = 000000$ $e(001) = 001100$ $e(010) = 010011$ $e(011) = 011111$

$e(100) = 100101$ $e(101) = 101001$ $e(110) = 110110$ $e(111) = 111010$

Decode the following words relative to a maximum likelihood decoding function.

- (i) 000101 (ii) 010101

- Q.5 (a) Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. [6M]

- (b) Use mathematical induction to show that $1+5+9+\dots+(4n-3) = n(2n-1)$ [6M]

- (c) Find the greatest lower bound and least upper bound of the set $\{3, 9, 12\}$ and $\{1, 2, 4, 5, 10\}$ if they exist in the poset $(Z^+, /)$. Where $/$ is the relation of divisibility. [8M]

- Q.6 (a) Let $A = \{1, 2, 3, 4\}$ and Let $R = \{(1,1) (1,2) (1,4) (2,4) (3,1) (3,2) (4,2) (4,3) (4,4)\}$. Find transitive closure by Warshall's algorithm. [6M]

- (b) Let $H = \{[0]_6, [3]_6\}$ find the left and right cosets in group Z_6 . Is H a normal subgroup of group of Z_6 . [6M]

- (c) Find the complete solution of the recurrence relation $a_n + 2a_{n-1} = n+3$ for $n \geq 1$ and with $a_0 = 3$ [8M]