

(3 Hours)

Max Marks: 80

- Note:**
1. Question No. 1 is compulsory.
 2. Out of remaining questions, attempt any three questions.
 3. Assume suitable additional data if required.
 4. Figures in brackets on the right hand side indicate full marks.

- (A) State Central limit theorem and give its significance (05)
 - (B) State the three axioms of probability. (05)
 - (C) State and explain Bayes Theorem. (05)
 - (D) Define Power spectral density and prove any two properties. (05)
- (A) Prove that if input to LTI system is w.s.s. then the output is also w.s.s. (10)
 - (B) In a factory, four machines A_1, A_2, A_3 and A_4 produce 35%, 10%, 25% and 30% of the items respectively. The percentage of defective items produced by them is 3%, 5%, 4% and 2%, respectively. An item is selected at random.
 - (i) What is the probability that the selected item will be defective?
 - (ii) Given that the item is defective what is the probability that it was produced by machine A_4 ?
- The joint probability density function of two random variables is given by (20)

$$f_{x,y}(x, y) = 15e^{-3x-3y} : x \geq 0, y \geq 0$$
 - i) Find the probability that $x < 2$ and $y > 0.2$.
 - ii) Find the marginal densities of x and y .
 - iii) Are x and y independent?
 - iv) Find $E(x/y)$ and $E(y/x)$.
- (A) A stationary process is given by $X(t) = 10 \cos [100t + \theta]$ where θ is a random variable with uniform probability distribution in the interval $[-\pi, \pi]$. Show that it is a wide sense stationary process. (10)
 - (B) Explain Strong and weak law of large numbers. (05)
 - (C) Write short notes on the following special distributions. (05)
 - i) Uniform distribution.
 - ii) Gaussian distribution.
- (A) Define discrete and continuous random variables by giving examples. Discuss the properties of distribution function. (10)
 - (B) A random variable has the following exponential probability density function: (10)

$$f(x) = Ke^{-|x|}$$
 Determine the value of K and the corresponding distribution function.
- (A) Suppose X and Y are two random variables. Define covariance and correlation of X and Y . When do we say that X and Y are (10)
 - (i) Orthogonal,
 - (ii) Independent, and
 - (iii) Uncorrelated?
 Are uncorrelated variables independent?
 - (B) State and prove Chapman-Kolmogorov equation. (10)

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