

(3 hours)

Total Marks:80

N.B:

1. Question No .1 is compulsory.
2. Answer any **three** questions from **Q. 2 to Q. 6**
3. Use of statistical tables permitted.
4. Figures to the right indicate full marks.

- 1) (a) A continuous random variable  $x$  has the pdf  $f(x) = kx^2e^{-x}$  where  $x \geq 0$ . Find  $k$ , its mean and variance. 5
- (b) State true or false with reasoning:  $2x+y=3$  and  $x=2y+3$  cannot be the lines of regression. 5
- (c) Find the relative maximum or minimum of the function  $z=x_1^2+x_2^2+x_3^2-6x_1-8x_2-10x_3$ . 5
- (d) Find the eigen values of  $\text{adj.}A$  and  $A^2-2A+I$  where  $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ . 5
- 2) (a) Obtain the rank correlation coefficient from the following data. 6
- X: 10 12 18 18 15 40
- Y: 12 18 25 25 50 25
- (b) The marks obtained by the students in Maths, Physics & Chemistry in an examination are normally distributed with the means 52, 50 & 48 and with standard deviations 10, 8 & 6 respectively. Find the probability that a student selected at random has secured a total of i) 180 or above ii) 135 or less. 6
- (c) Is the matrix  $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$  diagonalisable? If so, find the diagonal form and the transformation matrix. 8
- 3) (a) If  $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ , find  $A^{50}$ . 6
- (b) A die was thrown 132 times and the following frequencies were observed 6
- No. obtained : 1 2 3 4 5 6
- Frequencies : 15 20 25 15 29 28
- Test the hypothesis that the die is unbiased.
- (c) Use duality to solve the following linear programming problem. 8
- Minimise  $Z = 4x_1 + 3x_2 + 6x_3$  subject to
- $x_1 + x_3 \geq 2$ ;
- $x_2 + x_3 \geq 5$ ,  $x_1, x_2, x_3 \geq 0$ .

4) (a) A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165 cm and the SD is 10 cm?

(b) A transmission channel has a per digit error probability  $p=0.01$ . Calculate the probability of more than one error in 10 received digits using i) Binomial distribution ii) Poisson distribution.

(c) Evaluate  $\int_0^{2\pi} \frac{1}{3+2\cos\theta} d\theta$ .

5 .(a) Evaluate  $\int \frac{1}{z^3(z+4)} dz$  where C is the circle  $|z|=2$ .

(b) show that the matrix  $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$  is derogatory.

(c) Samples of 2 types of electric bulbs were tested for length of life and the following data were obtained

|          | Size | Mean  | SD  |
|----------|------|-------|-----|
| Sample 1 | 8    | 1234h | 36h |
| Sample 2 | 7    | 1036h | 40h |

Is the difference in the means sufficient to warrant that type 1 bulbs are superior to type 2 bulbs?

6 (a). Using the Big-M penalty method, solve the following L.P.P

Minimise  $Z=10x_1+3x_2$

subject to  $x_1+2x_2 \geq 3$

$x_1+4x_2 \geq 4$   $x_1, x_2 \geq 0$ .

(b) Use the Kuhn-Tucker conditions to solve the following N.L.P.P

Maximise  $Z=2x_1^2 - 7x_2^2 + 12x_1x_2$

Subject to  $2x_1+5x_2 \leq 98$   $x_1, x_2 \geq 0$

(c) Obtain Taylor's and Laurent's expansion for  $f(z)=\frac{z-1}{(z-3)(z+1)}$  indicating the regions of convergence.

\*\*\*\*\*