

Duration: 3 Hours

Total Marks: 80

N.B. : 1) Q.1. is compulsory.

2) Attempt any three from the remaining.

Q.1. a) Show that the set $\{e^x, xe^x, x^2e^x\}$ is linearly independent in $C^2(-\infty, \infty)$. (5)

b) Show that $\int_C \log z dz = 2\pi i$, where C is the unit circle in the z-plane. (5)

c) Find the projection of $u=(3,1,3)$ along and perpendicular to $v=(4,-2,2)$ (5)

d) Find the extremal of $\int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x) dx$ (5)

Q.2. a) If $A = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$, find e^A (6)

b) Evaluate $\int_0^\pi \frac{d\theta}{3 + 2\cos\theta}$ (6)

c) Find the singular value decomposition of $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ (8)

Q.3. a) Find the extremal of $\int_0^\pi (y'^2 - y^2) dx$ given $y(0) = 0, y(\pi) = 0$ (6)

b) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ and hence find A^{-1} & A^4 (6)

c) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the regions (i) $1 < |z-1| < 2$ (ii) $|z| < 1$ (8)

Q.4. a) Construct an orthonormal basis of R^3 using Gram Schmidt process to $S = \{(3,1),(2,3)\}$ (6)

b) Find the extremum of $\int_{x_0}^{x_1} (2xy + y'''^2) dx$. (6)

c) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy + 4xz - 2zy$ to canonical form and hence, find its rank, index and signature and value class. (8)

Q.5. a) Using Residue theorem evaluate $\int_C \frac{z^2}{(z-1)^2(z+1)} dz$ where C is $|z|=2$. (6)

b) Find the linear transformation $Y=AX$ which carries $X_1 = (1, 0, 1)'$, $X_2 = (1, -1, 1)'$, $X_3 = (1, 2, -1)'$ onto $Y_1 = (2, 3, -1)'$, $Y_2 = (3, 0, -2)'$, $Y_3 = (-2, 7, 1)'$ (6)

c) Check whether $V = \mathbb{R}^2$ is a vector space with respect to the operations

$$(x_1, 0) + (x_2, 0) = (x_1 + x_2, 0); k(x_1, 0) = (kx_1, 0) \quad (8)$$

Q.6.a) Obtain Taylor's series expansion for $f(z) = \frac{2z^3 + 1}{z(z+1)}$ about $z = i$ (6)

b) Let $W = \text{span} \left\{ (0, 1, 0), \left(\frac{-4}{5}, 0, \frac{3}{5} \right) \right\}$, Express $w = (1, 2, 3)$ in the form of $w = w_1 + w_2$ where

$$w_1 \in W \text{ \& } w_2 \in W^\perp \quad (6)$$

c) Using Rayleigh- Ritz method, solve the boundary value problem $I = \int_0^1 (2xy - y^2 - y'^2) dx$;

given $y(0) = y(1) = 0$ (8)
