Rules for Making Bode Plots

Term	Magnitude	Phase
Constant: K	$20 \cdot \log_{10}(\lvert K \rvert)$	K>0: 0° K<0: ±180°
Real Pole: $\dfrac{1}{\frac{s}{\omega_0}+1}$	• Low freq. asymptote at 0 dB • High freq. asymptote at -20 dB/dec • Connect asymptotic lines at ω_0,	• Low freq. asymptote at 0°. • High freq. asymptote at -90°. • Connect with straight line from $0.1 \cdot \omega_0$ to $10 \cdot \omega_0$.
Real Zero*: $\frac{s}{\omega_0}+1$	• Low freq. asymptote at 0 dB • High freq. asymptote at +20 dB/dec. • Connect asymptotic lines at ω_0.	• Low freq. asymptote at 0°. • High freq. asymptote at +90°. • Connect with line from $0.1 \cdot \omega_0$ to $10 \cdot \omega_0$.
Pole at Origin: $\frac{1}{s}$	• -20 dB/dec; through 0 dB at ω=1.	• -90° for all ω.
Zero at Origin*: s	• +20 dB/dec; through 0 dB at ω=1.	• +90° for all ω.
Underdamped Poles: $\dfrac{1}{\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1}$	• Low freq. asymptote at 0 dB. • High freq. asymptote at -40 dB/dec. • Connect asymptotic lines at ω_0. • Draw peak† at freq. ω_0, with amplitude $H(j\omega_0)=-20 \cdot \log_{10}(2\zeta)$	• Low freq. asymptote at 0°. • High freq. asymptote at -180°. • Connect with straight line from $\omega=\omega_0 \cdot 10^{-\zeta}$ to $\omega_0 \cdot 10^{\zeta}$
Underdamped Zeros*: $\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1$	• Low freq. asymptote at 0 dB. • High freq. asymptote at +40 dB/dec. • Connect asymptotic lines at ω_0. • Draw dip† at freq. ω_0, with amplitude $H(j\omega_0)=+20 \cdot \log_{10}(2\zeta)$	• Low freq. asymptote at 0°. • High freq. asymptote at +180°. • Connect with straight line from $\omega=\omega_0 \cdot 10^{-\zeta}$ to $\omega_0 \cdot 10^{\zeta}$
Time Delay: e^{-sT}	• No change in magnitude	• Phase drops linearly. Phase = $-\omega T$ radians or $-\omega T \cdot 180/\pi$°. On logarithmic plot phase appears to drop exponentially.

Notes:

ω_0 is assumed to be positive

* Rules for drawing zeros create the mirror image (around 0 dB, or 0°) of those for a pole with the same ω_0.

† We assume any peaks for $\zeta>0.5$ are too small to draw, and ignore them. However, for under damped poles and zeros peaks exists for $0<\zeta<0.707=1/\sqrt{2}$ and peak freq. is not exactly at, ω_0 (peak is at $\omega_{peak}=\omega_0\sqrt{1-2\zeta^2}$).

For n^{th} order pole or zero make asymptotes, peaks and slopes n times higher than shown. For example, a double (i.e., repeated) pole has high frequency asymptote at -40 dB/dec, and phase goes from 0 to –180°). Don't change frequencies, only the plot values and slopes.

Matlab Tools for Bode Plots

```
>> n=[1 11 10];                    %A numerator polynomial (arbitrary)
>> d=[1 10 10000 0];               %Denominator polynomial (arbitrary)
>> sys=tf(n,d)
Transfer function:
    s^2 + 11 s + 10
----------------------
s^3 + 10 s^2 + 10000 s

>> damp(d)                         %Find roots of den. If complex, show zeta, wn.
        Eigenvalue              Damping     Freq. (rad/s)
  0.00e+000                   -1.00e+000      0.00e+000
 -5.00e+000 + 9.99e+001i       5.00e-002      1.00e+002
 -5.00e+000 - 9.99e+001i       5.00e-002      1.00e+002

>> damp(n)                         %Repeat for numerator
Eigenvalue       Damping      Freq. (rad/s)
 -1.00e+000      1.00e+000      1.00e+000
 -1.00e+001      1.00e+000      1.00e+001

>> %Use Matlab to find frequency response (hard way).
>> w=logspace(-2,4);               %omega goes from 0.01 to 10000;
>> fr=freqresp(sys,w);
>> subplot(211);  semilogx(w,20*log10(abs(fr(:))));  title('Mag response, dB')
>> subplot(212);  semilogx(w,angle(fr(:))*180/pi);   title('Phase resp, degrees')

>> %Let Matlab do all of the work
>> bode(sys)

>> %Find Freq Resp at one freq.     %Hard way
>> fr=polyval(n,j*10)./polyval(d,j*10)
fr =   0.0011 + 0.0010i

>> %Find Freq Resp at one freq.     %Easy way
>> fr=freqresp(sys,10)
fr =   0.0011 + 0.0009i

>> abs(fr)
ans =    0.0014

>> angle(fr)*180/pi   %Convert to degrees
ans =   38.7107


>> %You can even find impulse and step response from transfer function.
>> step(sys)
>> impulse(sys)
```

```
>> [n,d]=tfdata(sys,'v')              %Get numerator and denominator.
n =
     0     1    11    10
d =
           1          10       10000           0

>> [z,p,k]=zpkdata(sys,'v')           %Get poles and zeros
z =
   -10
    -1
p =
         0
  -5.0000 +99.8749i
  -5.0000 -99.8749i
k =
     1
```

```
>> %BodePlotGui - Matlab program shows individual terms of Bode Plot.  Code at:
>> %   http://lpsa.swarthmore.edu/NatSci/Bode/BodePlotGui.html
>>
>> BodePlotGui(sys)
```