

QP Code : 4787

(3 Hours)
[Revised Course]

[Total Marks : 80]

- N.B.: 1) Question No.1 is compulsory.
2) Attempt any three from the remaining questions.
3) Assume suitable data if necessary.

1. (a) Determine the constants a, b, c, d if $f(z) = x^2 + 2axy + by^2 + i(dx^2 + 2cy + y^2)$ is analytic. 5

(b) Find a cosine series of period 2π to represent $\sin x$ in $0 \leq x \leq \pi$ 5

(c) Evaluate by using Laplace Transformation $\int_0^\infty e^{-3x} t \cos t \, dt$. 5

(d) A vector field is given by $\vec{F} = (x^2 + xy^2) \mathbf{i} + (y^2 + x^2 y) \mathbf{j}$. Show that \vec{F} is irrotational and find its scalar potential. Such that $\vec{F} = \nabla \phi$. 5

2. (a) Solve by using Laplace Transform 6

$$(D^2 + 2D + 5)y = e^{-t} \sin t, \text{ when } y(0) = 0, y'(0) = 1.$$

(b) Find the total work done in moving a particle in the force field $\vec{F} = 3xy \mathbf{i} - 5z \mathbf{j} + 10x \mathbf{k}$ along $x = t^2 + 1, y = 2t^2, z = t^3$ from $t=1$ and $t=2$. 6

(c) Find the Fourier series of the function $f(x) = e^{-x}, 0 < x < 2\pi$ and $f(x + 2\pi) = f(x)$. Hence deduce that the value of $\sum_{n=2}^\infty \frac{(-1)^n}{n^2 + 1}$. 8

3. (a) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \cdot \sin x$ 6

(b) Verify Green's theorem in the plane for $\oint (x^2 - y) dx + (2y^2 + x) dy$ Around the boundary of region defined by $y = x^2$ and $y = 4$. 6

(c) Find the Laplace transforms of the following. 8

i) $e^{-t} \int_0^t \frac{\sin u}{u} du$ ii) $t \sqrt{1 + \sin t}$

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- 4 (a) If $f(x) = C_1 Q_1(x) + C_2 Q_2(x) + C_3 Q_3(x)$, where C_1, C_2, C_3 constants and Q_1, Q_2, Q_3 are orthonormal sets on (a, b) , show that 6

$$\int_a^b [f(x)]^2 dx = C_1^2 + C_2^2 + C_3^2.$$

- (b) If $v = e^x \sin y$, prove that v is a Harmonic function. Also find the corresponding harmonic conjugate function and analytic function. 6

- (c) Find inverse Laplace transforms of the following. 8

i) $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ ii) $\frac{s+2}{s^2-4s+13}$

- 5 (a) Find the Fourier series if $f(x) = |x|$, $-k < x < k$ 6

Hence deduce that $\sum \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}$.

- (b) Define solenoidal vector. Hence prove that $\vec{F} = \frac{\vec{r} \times \vec{r}}{r^n}$ is a solenoidal vector 6

- (c) Find the bilinear transformation under which $1, i, -1$ from the z -plane are mapped onto $0, 1, \infty$ of w -plane. Further show that under this transformation the unit circle in w -plane is mapped onto a straight line in the z -plane. Write the name of this line. 8

- 6 (a) Using Gauss's Divergence Theorem evaluate $\iint_S \vec{F} \cdot d\vec{s}$ where $\vec{F} = 2x^2 y \hat{i} - y^2 \hat{j} + 4xz^2 \hat{k}$ and S is the region bounded by $y^2 + z^2 = 9$ and $x = 2$ in the first octant. 6

- (b) Define bilinear transformation. And prove that in a general, a bilinear transformation maps a circle into a circle. 6

- (c) Prove that $\int_{x=1/3}^{x=2/3} (x^{3/2}) dx = -\frac{2}{3} x^{-1/2} \Big|_{1/3}^{2/3} (x^{3/2})$. 8