Tut - 1

Even Roll Nos.

(1) $f(t) = t \quad 0<t<5$

$\quad\;\; = 0 \quad t>5$

Soln: $L(f(t)) = \int_0^\infty e^{-st} f(t)\, dt$ (1)

$= \int_0^5 t\, e^{-st}\, dt + \int_t^\infty 0\, e^{-st}\, dt$ (1)

$= \left[t \cdot \frac{e^{-st}}{-s} - 1 \cdot \frac{e^{-st}}{s^2} \right]_0^5$

$= \left(5 \cdot \frac{e^{-5t}}{-s} - \frac{e^{-5t}}{s^2} \right) - \left(0 - \frac{1}{s^2} \right)$

$= \frac{1}{s^2} - \frac{5 \cdot e^{-5t}}{s} - \frac{1}{s^2} e^{-5t}$ (3)

(2) $\cos 3t \cdot \cos 2t \cdot \cos t = \frac{1}{2} (2 \cos 3t \cdot \cos 2t) \cos t$

$= \frac{1}{2} [\cos 5t + \cos t] \cdot \cos t$ (1)

$= \frac{1}{4} [2 \cos 5t \cdot \cos t + 2 \cos t \cos t]$

$= \frac{1}{4} [\cos 6t + \cos 4t + 1 + \cos 2t]$ (1)

$L(\cos 3t \cdot \cos 2t \cdot \cos t) = \frac{1}{4} [L(\cos 6t) + L(\cos 4t) + L(1) + L(\cos 2t)]$

$= \frac{1}{4} \left[\frac{s}{s^2+6^2} + \frac{s}{s^2+4^2} + \frac{1}{s} + \frac{s}{s^2+2^2} \right]$

(3)

(3) $\int_0^{\infty} e^{-2t} \sin^3 t \, dt$

Comparing with:

$$\int_0^{\infty} e^{-st} f(t)\,dt = L(f(t)) \quad (1)$$

$$\therefore \int_0^{\infty} e^{-2t} \sin^3 t \, dt = L(\sin^3 t)_{s=2}$$

But $\sin^3 t = \dfrac{3\sin t - \sin 3t}{4}$ (1)

$$L(\sin^3 t) = \frac{1}{4}\left[3 \cdot L(\sin t) - L(\sin 3t)\right]$$

$$= \frac{1}{4}\left[\frac{3}{s^2+1^2} - \frac{3}{s^2+3^2}\right]$$

Put $s = 2$

$$\int_0^{\infty} e^{-2t} \sin^3 t \, dt = \frac{1}{4}\left[\frac{3}{5} - \frac{3}{13}\right]$$

$$= \frac{1}{4}\left[\frac{3 \cdot 13 - 15}{13 \cdot 5}\right]$$

$$= \frac{-1}{130}$$

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$\frac{(3)\,5}{13\,0}$

$= \frac{6}{65}$

(3)

(iv) **Statement**

If $L(f(t)) = \phi(s)$ then
$L(e^{at} f(t)) = \phi(s-a)$

$L(\sinh 4t \cdot t^4 \cdot e^{-2t}) = ?$

Now, $\sinh 4t = \dfrac{e^{4t} - e^{-4t}}{2}$

$e^{-2t} \cdot \sinh 4t = \dfrac{e^{-2t}}{2} (e^{4t} - e^{-4t})$

$e^{-2t} \cdot \sinh 4t = \dfrac{1}{2} (e^{2t} - e^{-6t})$

& $L(t^n) = \dfrac{\sqrt{n+1}}{s^{n+1}}$

$L(t^4) = \dfrac{\sqrt{4+1}}{s^{4+1}} = \dfrac{4!}{s^5} = \dfrac{4!}{s^5}$

$L(e^{-2t} \cdot t^4 \cdot \sinh 4t) = L\left[\dfrac{t^4}{2}(e^{2t} - e^{-6t})\right]$

$= \dfrac{1}{2} \left[L(t^4 \cdot e^{2t}) - L(e^{-6t} \cdot t^4)\right]$

Using FST.

$= \dfrac{1}{2} \left[\dfrac{4!}{(s-2)^5} - \dfrac{4!}{(s+6)^5}\right]$

Tut - 1

Odd Roll No.

(1) $f(t) = \sin 2t \qquad 0 < t < \pi$

$\qquad = 0 \qquad t > \pi$

Find $L(f(t))$

Soln: $L(f(t)) = \int_0^\infty e^{-st} \cdot f(t)\, dt$ (1)

$= \int_0^\pi e^{-st} \cdot f(t)\, dt + \int_\pi^\infty e^{-st} \cdot f(t)\, dt$

$= \int_0^\pi e^{-st} \cdot \sin 2t\, dt + 0$ (1)

$\left\{ \int e^{-ax} \cdot \sin bx\, dx = \frac{e^{-ax}}{a^2+b^2} (-a \sin bx - b \cos bx) \right\}$

$= \left\{ \frac{e^{-st}}{s^2+4} \left[-s \sin 2t - 2 \cos 2t \right] \right\}_0^\pi$

$= \frac{+1}{s^2+4} (-2)\, e^{-\pi s} + \frac{2}{s^2+4}$

$= \frac{2}{s^2+4} \left[1 - e^{-\pi s} \right]$ (3)

(2) $\sin t \cdot \cos 6t \cdot \cos 2t = \frac{1}{2} (2 \sin t \cdot \cos 6t) \cos 2t$

$= \frac{1}{2} \left[\sin 7t - \sin 5t \right] \cos 2t$ (1)

$= \frac{1}{4} \left[2 \sin 7t \cos 2t - 2 \sin 5t \cdot \cos 2t \right]$

$= \frac{1}{4} \left[\sin 9t - \sin 5t - \sin 7t + \sin 3t \right]$ (1)

$\therefore L[\sin t \cdot \cos 6t \cdot \cos 2t]$

$= \frac{1}{4}\left[L(\sin 9t) - L(\sin 5t) - L(\sin 7t) + L(\sin 3t)\right]$

$= \frac{1}{4}\left[\frac{9}{s^2+9^2} - \frac{5}{s^2+5^2} - \frac{7}{s^2+7^2} + \frac{3}{s^2+3^2}\right]$ (3)

(3) Evaluate integral $\int_0^\infty e^{-3t} \cdot \cos^3 t \, dt$

Sol. $\cancel{\cos 3\theta = -3\cos\theta + 4\cos^3\theta}$

$\cos^3\theta = \frac{1}{4}\left[\cos 3\theta + 3\cos\theta\right]$

comparing given integral with

$\int_0^\infty e^{-st} \cdot f(t)\, dt = L(f(t))$ (1)

$s = 3$

$f(t) = \cos^3 t$

$L(\cos^3 t) = L\left[\frac{1}{4}(\cos 3\theta + 3\cos(\theta))\right]$

$= \frac{1}{4}\left[L(\cos 3\theta) + L(3\cos\theta)\right]$ (1)

$L(\cos^3 t) = \frac{1}{4} \cdot \frac{s}{s^2+3^2} + \frac{3}{4} \cdot \frac{s}{s^2+1^2}$

$\therefore \int_0^\infty e^{-3t} \cos^3 t\, dt = \left[\frac{1}{4} \cdot \frac{s}{s^2+3^2} + \frac{3}{4} \cdot \frac{s}{s^2+1^2}\right]_{s=3}$

$= \frac{1}{4} \cdot \frac{3}{3^2+3^2} + \frac{3}{4} \cdot \frac{3}{3^2+1^2}$

$= \frac{1}{24} + \frac{9}{40} = \frac{4}{15}$ (3)

$= \frac{5 + 3(9)}{120} = \frac{[illegible]}{120}$

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First shifting theorem:

(4) If $\mathcal{L}[f(t)] = \phi(s)$

then $\mathcal{L}[e^{at} f(t)] = \phi(s-a)$ (1)

$\mathcal{L}[(t+2)^2 \cdot e^t] = ?$

$(t+2)^2 = t^2 + 4t + 4$

$L(t+2)^2 = L(t^2 + 4t + 4)$

$= L(t^2) + 4\,L(t) + 4\,L(1)$

$= \frac{\Gamma 3}{s^3} + 4 \cdot \frac{\Gamma 2}{s^2} + 4 \cdot \frac{1}{s}$

$= \frac{2!}{s^3} + \frac{4 \cdot 1!}{s^2} + \frac{4 \cdot 1}{s}$

$= \frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}$ (3)

$L(t^n) = \frac{\Gamma n+1}{s^{n+1}}$

Using: FST $s \to s-2$

$L(e^t (t+2)^2) = \frac{2}{(s-2)^3} + \frac{4}{(s-2)^2} + \frac{4}{s-2}$