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Tut test 3

odd Roll Nos.

(1) $f(x) = 1 + \frac{2x}{\pi} \qquad -\pi < x < 0.$

8M

$= 1 - \frac{2x}{\pi} \qquad 0 < x < \pi.$

(2) $f(x) = \left(\frac{\pi - x}{2}\right)^2 \quad ; (0, 2\pi)$

Hence P.T.

(i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$

C(2M).

(ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} \dots = \frac{\pi^2}{12}$

Soln: $a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x)\,dx = \frac{1}{2\pi} \int_0^{2\pi} \frac{(\pi - x)^2}{4}\,dx$

$= \frac{1}{8\pi} \left[\frac{(\pi - x)^3}{-3}\right]_0^{2\pi}$

(4)

$= \frac{\pi^2}{\cancel{24}} \cdot 2 = \frac{\pi^2}{12}$

$a_n = \frac{1}{\pi} \int_0^{2\pi} \frac{(\pi - x)^2}{4} \cdot \cos nx\,dx.$

$a_n = \frac{1}{4\pi}\left[(\pi - x)^2 \cdot \left(\frac{\sin nx}{n}\right) - 2(\pi - x)(-1)\left(-\frac{\cos nx}{n^2}\right) + 2(-1)(-1)\left(\frac{-\sin nx}{n^3}\right)\right]_0^{2\pi}$

$a_n = \frac{1}{n^2}.$

(5)

$b_n = \frac{1}{\pi} \int_0^{2\pi} \frac{(\pi - x)^2}{4} \cdot \sin nx\,dx$

$= \frac{1}{4\pi}\left[(\pi - x)^2 \cdot \frac{(-\cos nx)}{n} - 2(\pi - x)(-1) \cdot \frac{(-\sin nx)}{n^2} + 2(-1)(-1)\frac{\cos nx}{n^3}\right]_0^{2\pi}$

$= 0,$

(5)

$$\left(\frac{\pi-x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx$$

Put $x=0$

$$\frac{\pi^2}{4} = \frac{\pi^2}{12} + \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$$

$$\therefore \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}.$$ (3)

Put $x=\pi$

$$0 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos n\pi$$

$$0 = \frac{\pi^2}{12} + \frac{1}{1^2}\cos \pi + \frac{1}{2^2}\cos 2\pi + \frac{1}{3^2}\cos 3\pi + \ldots$$

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{12}$$ (3)

Even Roll Nos.

$$f(x) = 0 \qquad -\pi < x \le 0$$
$$= x \qquad 0 \le x \le \pi.$$

Hence deduce that

(i) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

(ii) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots = \frac{\pi}{4}$

Soln: $a_0 = \frac{1}{2\pi} \int_0^{\pi} x\,dx = \frac{\pi}{4}.$ (4)

$a_n = \frac{1}{\pi} \int_0^{\pi} x \cos nx \, dx = \frac{\pi}{2}$

$= \frac{1}{\pi} \left[x \cdot \frac{\sin nx}{n} - \frac{(1)(-\cos nx)}{n^2} \right]_0^{\pi}$

$= \frac{1}{\pi} \left[\frac{1}{n^2} (\cos n\pi - 1) \right]$

now $a_n = \frac{2}{n^2 \pi} (-1)^n$ - n odd (5)

$= 0$ n even

$b_n = \frac{1}{\pi} \int_0^{\pi} x \sin nx$

$= \frac{1}{\pi} \left[x \cdot \frac{(-\cos nx)}{n} - (1) \cdot \frac{(-\sin nx)}{n^2} + (0) \frac{\cos nx}{n^3} \right]_0^{\pi}$

$= \frac{1}{\pi} \left[\left[\frac{-\pi \cos n\pi}{n} + \frac{\cos n\pi}{n^3} \right]_0^{\pi} - \frac{1}{n^3} \right]$

$= \frac{1}{\pi} \cdot \frac{-\pi \cdot \cos n\pi}{n}$

$b_n = \frac{-(-1)^n}{n}$ (5)

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos(2n-1)x$$

$$- \sum \frac{(-1)^n}{n} \sin nx$$

(i) put $x = 0$

$$f(0) = \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

$$0 = \frac{\pi}{4} - \frac{2}{\pi} \left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \right]$$ (3)

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

$\frac{5\pi}{2} = 2\pi + \frac{1}{\pi}$

(ii) put $x = \frac{\pi}{2}$

$$f(\pi/2) = \frac{\pi}{4} - \sum_{n=1}^{\infty} \sin \frac{n\pi}{2} \cdot \frac{(-1)^n}{n}$$

$$\frac{\pi}{2} = \frac{\pi}{4} - \left\{ -1 + \frac{1}{2}(0) - \frac{1}{3}(-1) + 0 - \frac{1}{5}(+1) \cdots \right.$$

$$\frac{\pi}{2} - \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} \cdots$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} \cdots$$ (3)