Int test : 2

Solution.

Odd Roll Nos.

2.8.19

① $L^{-1}\left[\frac{s+2}{s^2+4s+13}\right] = L^{-1}\left[\frac{s+2}{s^2+4s+4+9}\right]$

$= L^{-1}\left[\frac{s+2}{(s+2)^2+3^2}\right]$ (2)

By FST Put $s \to s-2$

$= e^{-2t}\, L^{-1}\left[\frac{s}{s^2+3^2}\right]$ (2)

$= e^{-2t} \cdot \cos 3t$ (2)

② $L^{-1}\left[\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right]$

$\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)} = \frac{x+3}{(x+2)(x+5)}$

Put $s^2+2s = x$

$= \frac{A}{x+2} + \frac{B}{x+5}$

$A(x+5) + B(x+2) = x+3$

Put $x = -5$ $\quad B(-3) = -2 \Rightarrow B = 2/3$

$x = -2$ $\quad A(3) = 1 \Rightarrow A = 1/3$

$= \frac{1}{3} \cdot \frac{1}{x+2} + \frac{2}{3} \cdot \frac{1}{x+5}$

$= \frac{1}{3} \cdot \frac{1}{s^2+2s+2} + \frac{2}{3} \cdot \frac{1}{s^2+2s+5}$

$= \frac{1}{3} \cdot \frac{1}{[(s+1)^2+1]} + \frac{2}{3} \cdot \frac{1}{[(s+1)^2+2^2]}$

$$L^{-1}\left[\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right] =$$

$$\frac{1}{3} L^{-1}\left[\frac{1}{(s+1)^2+1}\right] + \frac{2}{3} L^{-1}\left[\frac{1}{(s+1)^2+2^2}\right]$$

Using FST.

$$= \frac{1}{3} e^{-t} L^{-1}\left[\frac{1}{s^2+1}\right] + \frac{2}{3} e^{-t} L^{-1}\left[\frac{1}{s^2+2^2}\right]$$

$$= \frac{1}{3} e^{-t} \cdot \sin t + \frac{\cancel{2}}{3} e^{-t} \cdot \frac{1}{\cancel{2}} \sin 2t$$

$$= \frac{1}{3} e^{-t} [\sin t + \sin 2t]$$

(3) $\mathcal{L}^{-1}\left[\frac{s}{(s^2+4)(s^2+9)}\right]$ using convolution thm

$$\phi_1(s) = \frac{1}{s^2+4} \; ; \quad \phi_2(s) = \frac{s}{s^2+9}$$

$$\mathcal{L}^{-1}[\phi_1(s)] = \frac{1}{2} \sin 2t = f_1(t)$$

$$\mathcal{L}^{-1}[\phi_2(s)] = \cos 3t = f_2(t)$$

$$\mathcal{L}^{-1}[\phi_1(s) \cdot \phi_2(s)] = f_1(t) * f_2(t)$$

$$= \int_0^t f_1(u) \cdot f_2(t-u) \, du$$

$$= \int_0^t \frac{1}{2} \sin 2u \cdot \cos 3(t-u) \, du$$

$$= \frac{1}{4} \int_0^t \sin(\cancel{2u} + 3t - \cancel{3u}) + \sin(2u - 3t + 3u) \, du$$

$$= \frac{1}{4} \int_0^t \sin(3t - u) + \sin(5u - 3t) \, du$$

$$= \frac{1}{4}\left[-\cos(3t-4)(-1) + \cos\left(\frac{54-3t}{5}\right)(4)\right]$$

$$= \frac{1}{4}\left[\left(\cos 2t - \frac{\cos 2t}{5}\right) - \left(\cos(3t) - \frac{\cos 3t}{5}\right)\right]$$

$$= \frac{1}{4}\left[\frac{4\cos 2t}{5} - \frac{4\cos 3t}{5}\right]$$

$$= \frac{1}{5}\left[\cos 2t - \cos 3t\right]$$

Tut 2 (Even Roll Nos.)

Solution.

(1) $L^{-1}\left[\frac{2s}{[2s-1]^2}\right]$.

$$L^{-1}\left[\frac{2s}{(2s-1)^2}\right] = L^{-1}\left[\frac{2s}{4(s-\frac{1}{2})^2}\right] \quad (1)$$

Using FST. $s \to s+\frac{1}{2}$

$$= e^{\frac{1}{2}t} L^{-1}\left[\frac{2(s+\frac{1}{2})}{4s^2}\right] \quad (1)$$

$$= \frac{e^{\frac{t}{2}}}{2} L^{-1}\left[\frac{1}{s} + \frac{1}{2s^2}\right]$$

$$= \frac{e^{t/2}}{2}\left[L^{-1}\left[\frac{1}{s}\right] + L^{-1}\left[\frac{1}{2s^2}\right]\right] \quad (2)$$

$$= \frac{e^{t/2}}{2}\left[1 + \frac{t}{2}\right] \quad (2)$$

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(2) Find $L^{-1}\left[\dfrac{s^2}{s^4+9s^2+20}\right]$

$$\frac{s^2}{(s^2+5)(s^2+4)} = \frac{x}{(x+5)(x+4)} = \frac{A}{x+5} + \frac{B}{x+4} \quad (1)$$

Let $s^2 = x$

$$A(x+4) + B(x+5) = x$$

$x = -4$ $\quad B(1) = -4 \Rightarrow B = -4$

$x = -5$ $\quad A(-1) = -5 \Rightarrow A = 5$

$$\frac{s^2}{(s^2+5)(s^2+4)} = \frac{5}{s^2+5} - \frac{4}{s^2+4} \quad (1)$$

$$L^{-1}\left[\frac{s^2}{(s^2+5)(s^2+4)}\right] = 5\,L^{-1}\left[\frac{1}{s^2+5}\right] - 4\,L^{-1}\left[\frac{1}{s^2+4}\right]$$

[3]

$$= 5 \cdot \frac{1}{\sqrt{5}} \sin\sqrt{5}t - 4 \cdot \frac{1}{2} \sin 2t$$

$$= \sqrt{5}\sin\sqrt{5}t - 2\sin 2t. \quad [2]$$

(3) $L^{-1}\left[\dfrac{s^2+s}{(s^2+1)(s^2+2s+2)}\right]$

$$\frac{s^2+s}{(s^2+1)(s^2+2s+2)} = \frac{s(s+1)}{(s^2+1)((s+1)^2+1)} = \phi_1(s).\,\phi_2(s)$$

Let $\phi_1(s) = \dfrac{s}{s^2+1}$; $\phi_2(s) = \dfrac{s+1}{(s+1)^2+1}$ (1)

$L^{-1}[\phi_1(s)] = \cos t$; $L^{-1}[\phi_2(s)] = e^{-t}.\cos t$
$= f_1(t)$

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$$\mathcal{L}^{-1}[\phi_1(s)\phi_2(s)] = f_1(t) * f_2(t)$$

$$= \int_0^t f_1(u)\cdot f_2(t-u)\, dt \qquad (1)$$

$$= \int_0^t f_1(t-u)\cdot f_2(u)\, dt$$

$$= \frac{1}{2}\int_0^t 2\cos(t-u)\cdot e^{-u}\cdot \cos u \, du$$

$$= \frac{1}{2}\int_0^t [\cos((t-u)+u) + \cos(t-u-u)]\, e^{-u} du$$

$$= \frac{1}{2}\left\{\int_0^t e^{-u}[\cos t]\, du + \int_0^t e^{-u}\cdot \cos(t-2u) du\right. \qquad (2)$$

$$= \frac{1}{2}\cos t\left[\frac{e^{-u}}{-1}\right]_0^t + \left[\frac{e^{-u}}{1^2+2^2}\left(-\cos(t-2u) + (-2)\sin(t-2u)\right)\right]_0^t$$

$$= \frac{-1}{2}\cos t\,[e^{-t}-1] + \frac{-e^{-t}}{5}(\cos t - 2\sin t)$$

$$+ \frac{1}{5}(\cos t + \sin t)$$

(2)