S.E. (EXTC) / AM - III

complex Variables.

1. Check whether the following functions are analytic:

a) $\sin z$ (b) $\log z$ (c) $e^{i\bar{z}}$ (d) $\frac{\bar{z}}{z}$

e) $z \cdot \bar{z}$ (f) z^4 (g) $z^2 + 2z$

2. a) If $f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$ is analytic. Find constants a, b, c, d, e

b) If $f(z) = (ax^2 + by^2 + 5x) + i(cy + bxy)$ is analytic. Find a, b, c.

3. Show that ∄ analytic function ~~whose~~

(a) whose real part is $x^2 + xy^2$

(b) whose imaginary part is $x^3 + y^2$

c) whose real part is $\left(r - \frac{a^2}{r^2}\right)\sin\theta$.

4. Find the analytic function $f = u + iv$ if

(a) $u = e^x(\cos y - \sin y)$

(b) $u = x^3 - 3x^2y - 3y^2 + 3x^2y$

(c) $v = e^x(x \sin y + y \cos y)$

(d) $v = \frac{x}{x^2+y^2} + \cosh x \cdot \cos y$

(e) $u = r^3 \sin 3\theta$

(f) $u = r \cos\theta$

(g) $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$

(h) $3u + 2v = y^2 - x^2 + 16xy$

(i) $u - v = (x - y)(x^2 + 4xy + y^2)$

(j) $u + v = \frac{2x}{x^2+y^2}$

(5) Show that following functions are harmonic. Also find their harmonic conjugates.

(a) $u = x^2 - y^2$ (b) $v = \dfrac{y}{x^2+y^2}$

(c) $u = \log r$ (d) $u = e^x \cos y + x^3 - 3xy^2$

6) Find orthogonal trajectories of the curves given by:

a) $x^2 - y^2 = c$ (b) $e^x \cos y - xy = 0$

(c) $3x^2y + 2x^2 - y^3 - 2y^2 = c$

(7) Find the image of following under $w = \frac{1}{z}$.

(a) $|z - 3i| = 3$ (b) $2 \le x \le 4$ (c) $|z-3| = 5$

(d) $y - x = 0$ (e) $0 \le y \le \frac{1}{4}$ (f) $y - x + 1 = 0$.

Show the regions graphically.

8) Show that under transformation $w = \frac{1}{z}$, the interior (exterior) of circle $|z| = 1$ is mapped onto the exterior (interior) of $|w| = 1$.

9) Find the image of $|z| = 1$ under $w = \dfrac{5 - 4z}{4z - 2}$

(10) Show that $w = \dfrac{2z+3}{z-4}$ transforms circle $x^2 + y^2 = 4x$ into the line $4u + 3 = 0$

11) $w = \dfrac{i - z}{i + z}$. Find image of $|z| < 1$

(12) Find the image of x-axis, y-axis, under transformation $w = \dfrac{1}{2z+1}$

(13) Find fixed points of bilinear transformation

(a) $w = \dfrac{1 + 3iz}{3i + z}$ (b) $w = \dfrac{2z - 2 + iz}{i + z}$ (c) $w = \dfrac{1}{z + i}$

(2)

(14) Find the bilinear transformation which maps:

(a) $z = -1, 1, \infty$ onto the points $w = -i, -1, i$

(b) $z = 2, 1, 0$ " " $w = 1, 0, i$

(c) $z = 0, 1, \infty$ " " $w = -5, -1, 3$

(d) $z = i, -1, 1$ " " $w = 0, 1, \infty$

(15) Show that $w = i\left(\frac{1-z}{1+z}\right)$ transforms the circle $|z|=1$ onto the real axis of the w-plane and the interior of the circle $|z|<1$ onto the upper half of the w-plane.