X. I. E. QUESTION BANK ON LAPLACE TRANSFORM

Mahim, Mumbai

Date :

I. Find the Laplace transform of the following:

1. $4t^2 + \sin t + e^{-2t}$
2. $(\sin 2t - \cos 2t)^2$
3. $\cos(\omega t - b)$
4. $\cosh^5 t$
5. $\sin^5 t$
6. $e^{4t}\cosh t$
7. $e^{3t}\, t^4$
8. $\sinh\frac{t}{2}\sin\frac{\sqrt{3}}{2}t$
9. $e^{-4t}\sinh t \sin t$
10. $(t+2)^2 e^t$
11. $(1 + te^{-t})^3$
12. $t^2 e^t \sin t$
13. $t\left(\frac{\sin t}{e^t}\right)^2$
14. $t\sin 2t \cosh t$
15. $(t^2 - 3t + 2)\sin 3t$
16. $e^{-3t}\frac{\sin t}{t}$
17. $\frac{1-\cos t}{t}$
18. $\left(\frac{\sin 2t}{\sqrt{t}}\right)^2$
19. $\left(\frac{\sin t}{t}\right)^2$
20. $\int_0^t u\cos^2 u\, du$
21. $\int_0^t u e^{-3u}\cosh^2 2u\, du$
22. $\int_0^t \frac{\sin u}{e^u\, u}\, du$
23. $e^{-t}\int_0^t \frac{\sin u}{u}\, du$
24. $t\int_0^t e^{-2u}\sin 4u\, du$
25. $t^{-1}\int_0^t e^{-3u}\sin 3u\, du$
26. $e^{-4t}\int_0^t u\sin 4u\, du$
27. $\cosh t \int_0^t e^u \sinh u\, du$
28. $\sin\sqrt{t}$
29. $\frac{\cos\sqrt{t}}{\sqrt{t}}$
30. $\frac{1}{\sqrt{\pi t}} + 6^{2t}$
31. $f(t) = \begin{cases} t+1 & 0 \le t \le 2 \\ 3 & t > 2 \end{cases}$
32. $f(t) = \begin{cases} \sin 2t & 0 < t < \pi \\ 0 & t > \pi \end{cases}$

II 1. If $\operatorname{erf} x = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$, find $L(e^{-3t}\operatorname{erf}\sqrt{t})$

2. If $\operatorname{erf} x + \operatorname{erf}_c x = 1$, find $L(\operatorname{erf}_c \sqrt{t})$

3. If $J_0(t) = \sum_{r=0}^{\infty} \frac{(-1)^r}{(r!)^2}\left(\frac{t}{2}\right)^{2r}$, find $L(J_0(t))$

III. Find the Laplace Transform of

1. $e^{-2t}\operatorname{erf}\sqrt{t}$
2. $e^{-5t}\operatorname{erf}(2\sqrt{t})$
3. $t\operatorname{erf}\sqrt{t}$
4. $e^{t}\operatorname{erf}_c\sqrt{t}$
5. $t\,J_0(2t)$
6. $e^{-3t}\,t\,J_0(4t)$

IV 1. If $\int_0^{\infty} e^{-2t}\sin(t+\alpha)\cos(t-\alpha)\,dt = \frac{3}{8}$, then find α.

2. If $L(f(t)) = \frac{8(3-s)}{s^2-6s+25}$, find $L(f(2t))$, $L(f(\frac{t}{2}))$.

3. If $L(f(t)) = \frac{2}{s^3}e^{-s}$, find $L(f(\frac{t}{3}))$, $L(f(3t))$

V. Evaluate the following integrals:

1. $\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t}\,dt$
2. $\int_0^{\infty} \frac{e^{-t}\sin t}{t}\,dt$
3. $\int_0^{\infty} e^{-t}\left(\int_0^t u\cos^2 u\,du\right)dt$
4. $\int_0^{\infty} e^{-t}\left(t\int_0^t e^{-4u}\cos u\,du\right)dt$
5. $\int_0^{\infty} e^{-t}(t^2-3t+5)\,dt$
6. $\int_0^{\infty} e^{-t}\,t\sqrt{1+\sin t}\,dt$

X. I. E.
Mahim, Mumbai

SE (IT) SEM - III AM - III

QUESTION BANK ON

INVERSE LAPLACE TRANSFORM

I. Find the inverse Laplace transform of

1. $\frac{1}{(s+1)^2} + \frac{s}{(2s+1)^2}$

2. $\frac{s-2}{s^2+4s+5} + \frac{2s}{s^2+4}$

3. $\frac{2s^2-4}{(s+1)(s-2)(s-3)}$

4. $\frac{1}{s(s+1)^2}$

5. $\frac{1}{s^2(s+3)^2}$

6. $\frac{1}{(s+3)(s-2)^4}$

7. $\frac{s^2}{(s+4)^3}$

8. $\frac{5s^2-7s+17}{(s-1)(s^2+4)}$

9. $\frac{2s^3-s^2-1}{(s+1)^2(s^2+1)^2}$

10. $\frac{5s+3}{(s-1)(s^2+2s+5)}$

11. $\frac{s^2}{(s^2+4)(s^2-9)}$

12. $\frac{s}{(s^2+4)(s^2-9)}$

13. $\frac{(s^2+4s+7)}{(s^2+4s+5)(s^2+4s+20)}$

14. $\frac{s+2}{(s^2+4s+8)(s^2+4s+13)}$

15. $\frac{2s}{s^4+4}$

16. $\frac{s}{s^4+s^2+1}$

17. $\frac{1}{s^3+1}$

18. $\frac{s}{(s^2-2s+2)(s^2+2s+2)}$

19. $\dfrac{s^2+s}{(s^2+1)(s^2+2s+2)}$

20. $\dfrac{s}{s^4+8s^2+16}$

21. $\log \dfrac{s^2-4}{(s-3)^2}$

22. $\dfrac{1}{s}\log\left(1+\dfrac{1}{s^2}\right)$

23. $\log\left(\dfrac{s^2+1}{s(s+1)}\right)$

24. $s\log\left(\dfrac{s+1}{s-1}\right)$

25. $2\tanh^{-1}s$

26. $\tan^{-1}\left(\dfrac{2}{s^2}\right)$

27. $\cot^{-1}(s+1)$

28. $\tan^{-1}\left(\dfrac{s}{2}\right)$

29. $\tan^{-1}\left(\dfrac{2}{s}\right)$

30. $\log\left(\dfrac{s^2+16}{\sqrt{s+3}}\right)$

II. Find the Laplace transform

1. $f(t)=\begin{cases} t & 0<t<\pi \\ \pi-t & \pi<t<2\pi \end{cases}$, $f(t+2\pi)=f(t)$

2. $f(t)=t \quad 0<t<1$
   $f(t+1)=f(t)$

3. $f(t)=t^2 \quad 0<t<2$ where f is periodic function with period 2.

III. Express the following functions in terms of Heaviside unit step function and hence find the Laplace transform :

1. $f(t)=\begin{cases} 0 & 0<t<3 \\ (t-3)^4 & t>3. \end{cases}$

2. $f(t)=\begin{cases} 0 & 0<t<2 \\ 1-2t-t^2 & t>2 \end{cases}$

3. $f(t)=\begin{cases} 2t & 0<t<1 \\ 3t^2 & t>1 \end{cases}$

4) $f(t)=\begin{cases} \cos t & 0<t<\pi \\ \cos 2t & \pi<t<2\pi \\ \cos 3t & 2\pi<t \end{cases}$

5) $f(t)=\begin{cases} 1 & 0<t<1 \\ 2 & 1<t<2 \\ 3 & 2<t<3 \\ \vdots & \vdots \end{cases}$ (Staircase function)

IV. Find the inverse laplace transform of

(1) $\dfrac{e^{-s}}{(s+1)^2}$ (3) $\dfrac{e^{-s}(1+\sqrt{s})}{s^3}$ (5) $\dfrac{e^{-4s}}{\sqrt{2s+7}}$

(2) $\dfrac{e^{-\pi s}}{s^2+9}$ (4) $\dfrac{se^{-s/2}+\pi e^{-s}}{s^2+\pi^2}$ (6) $e^{-3s}\cos 3 + \dfrac{s}{s+a}$

V. Using Laplace transform solve the following differential equations :

1 $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} - 3y = \sin t$ $y(0) = 0$, $y'(0) = 0$

2. $(D^2 - 2D - 8)y = 4$ $y(0) = 0$ and $y'(0) = 1$

3. $\dfrac{d^2y}{dt^2} + 9y = 18t$ $y(0) = 0$, $y(\frac{\pi}{2}) = 0$

4. $(D^2 + 3D + 2)y = 2(t^2 + t + 1)$ $y(0) = 2$, $y'(0) = 0$

5. $\dfrac{d^2y}{dt^2} + y = t$ $y(0) = 1$, $y'(0) = 0$

6. $D^2 - D - 2)y = 20\sin 2t$ $y(0) = 1$, $y'(0) = 2$

7. $\dfrac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$ $y(0) = 1$

8. $y + \int_0^t y\,dt = 1 - e^{-t}$

SE(IT) SEM-III APPLIED MATHEMATICS-III

QUESTION BANK ON

COMPLEX VARIABLE

1. Check whether the following functions are analytic :

a) $z\bar{z}$ b) $\sin \bar{z}$ c) e^{iz}

2. Use CR eqn in polar form to check the analyticity of

a) z^4 b) $z/\bar{z}$ c) $\log z$

3. If $f(z) = ax^2+by^2+5x+i(cy+6xy)$ is analytic, find a, b, c

4. Show that there does not exist an analytic function

a) whose real part is x^2+y^2

b) whose imaginary part is x^3+xy^2

5. Find the analytic function $f=u+iv$ if

a) $u = x^2-y^2-5x+y+2$

b) $u = e^{-x}(x\sin y - y\cos y)$

c) $u = \dfrac{\sinh 2x}{\cosh 2y - \cos 2x}$

d) $v = \dfrac{x}{x^2+y^2} + \cosh x \cos y$

e) $v = \log\sqrt{x^2+y^2}$

f) $u-v = (x-y)(x^2+4xy+y^2)$

g) $u+v = e^{x}(\cos y + \sin y)$

h) $3u+2v = \dfrac{x}{x^2+y^2}$

i) $u = r^2\cos 2\theta - r\sin\theta + 2$

j) $v = r^n \cos n\theta$

6. Show that the following functions are harmonic and find their harmonic conjugates :

a) $e^{x}\cos y + x^3 - 3xy^2$ b) $e^{2x}(x\cos 2y - y\sin 2y)$

7. Find the orthogonal trajectories of the curves given by.

a) $3x^2y + 2x^2 - y^3 - 2y^2 = c$ b) $e^{x}\cos y - xy = c$

c) $x^3y - y^3x = C$

8. Find the image of the circle $|z| = k$ under the transformation $w = z + 4 + 3i$.

9. Find the image of the semi-infinite strip $x > 0$, $1 < y < 3$ under the transformation $w = iz + 2$. Show the region graphically.

10. Find the image of the following under $w = 1/z$.

a) $x^2 + y^2 = 2x$
b) $|z - 3i| = 3$
c) $x = y$
d) $y = x + \frac{1}{2}$
e) $x + y = 1$
f) $\frac{1}{4} \leq y \leq \frac{1}{2}$
g) $0 \leq y \leq \frac{1}{4}$
h) $|z| = 2$
i) $|z - 1| = 1$
j) $|z| < 1$
k) $|z| > 1$
l) $y > c$ $\quad c > 0$
m) $x < c$ $\quad c > 0$
n) $x^2 - y^2 = 1$

11. Find the fixed points of

a) $\dfrac{2z - 2 + iz}{i + z}$ $\qquad$ b) $\dfrac{3z - 5i}{iz - 1}$

12. Find the image of y-axis under the transformation $w = \dfrac{1}{2z + 1}$

13. Find the image of real axis under the transformation $w = \dfrac{1}{z + i}$. Also find the fixed points of the transformation.

14. Find the bilinear transformation which maps

a) $z = 1, i, -1$ onto $w = i, 0, -i$
b) $z = 1, i, -1$ onto $w = 0, 1, \infty$
c) $z = 2, i, -2$ onto $w = 1, i, -1$