Laplace Transform

(i) First Shifting Theorem
(ii) Multiplication by 't'
(iii) Division by 't'

Find the Laplace Transform of the following:

1. $t e^{-t} \cosh 2t$
2. $t e^{-4t} \sin 3t$
3. $t^2 e^{-t} \sin 4t$
4. $(t \sin^3 t)/e^{3t}$
5. $t^5 \cosh t$
6. $t e^{-2t} \cos^2 t$
7. $t \sqrt{1+\sin t}$
8. $t e^{3t} \operatorname{erf} \sqrt{t}$

1. $\frac{1}{t}\left(e^{2t} \sin^3 t\right)$
2. $\frac{1-\cos t}{t^2}$
3. $\frac{\cosh t \sin^2 2t}{t}$
4. $\frac{\cos at - \cos bt}{t}$
5. $\frac{\sinh at}{t}$
6. $\frac{e^{-at} - \cos at}{t}$

Laplace Transform

(iv) Laplace transform of Integrals.
(v) Change of Scale
(vi) Heaviside's Unit Step Function, Dirac-delta function, Periodic Functions.

Find the Laplace transform of the following:

1. $\int_0^t e^{-2t} t^3 \, dt$

2. $\int_0^t t e^{-4t} \sin 3t \, dt$

3. $e^{-4t} \int_0^t t \sin 3t \, dt$

4. $e^{-t} \int_0^t \frac{\sin t}{t} \, dt$

5. $\int_0^t \int_0^t \int_0^t t \sin t \, dt$

6. $\cosh t \int_0^t e^{u} \cosh u \, du$

7. $\int_0^t \frac{1-e^{-u}}{u} \, du$

8. $\int_0^t u^2 \sin u \, du$

1. If $L(f(t)) = \frac{8(3-s)}{s^2 - 6s + 25}$, find $L(f(2t))$

2. If $L(f(t)) = \frac{2}{s^3} e^{-s}$, find $L(f(\frac{t}{3}))$

3. If $L(J_0(t)) = \frac{1}{\sqrt{1+s^2}}$, find $L(t e^{-3t} J_0(2t))$

Express the following functions in terms of Heaviside's Unit Step function and hence find the Laplace transform

$$f(t) = \begin{cases} \cos t & t > \pi \\ 0 & t < \pi \end{cases}$$

$$f(t) = \begin{cases} (t-2)^5 & t > 2 \\ 0 & 0 < t < 2 \end{cases}$$

$$f(t) = \begin{cases} t^2 & 0 < t < 1 \\ 4t & t > 1 \end{cases}$$

$$f(t) = \begin{cases} \sin 2t & 2\pi < t < 4\pi \\ 0 & \text{otherwise.} \end{cases}$$

Find the Laplace Transform of the following periodic function

$$f(t) = 2t \quad 0 < t < 4$$
$$f(t+4) = f(4)$$

$$f(t) = \begin{cases} a \sin \omega t & 0 < t < \pi/\omega \\ 0 & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$$

$$f(t) = f\left(t + \frac{2\pi}{\omega}\right)$$

$$f(t) = e^{2t} \quad 0 < t < 2\pi$$
$$f(t) = f(t + 2\pi)$$

INVERSE LAPLACE TRANSFORM

I. Find the inverse Laplace transform of :

i) $\frac{s+29}{(s+4)(s^2+9)}$ ii) $\frac{s}{(s^2+9)(s^2-4)}$ iii) $\frac{s^2+s}{(s^2+1)(s^2+2s+2)}$

iv) $\frac{1}{s^3+1}$ v) $\frac{2s}{s^4+4}$ vi) $\frac{s}{s^4+s^2+1}$

II. Solve using Laplace Transforms :

$\frac{dy}{dt} + 2y = e^{-3t}$ $y(0) = 1$

$\frac{d^2y}{dt^2} + 2y - 3y = 0$ $y(0) = 0$, $y'(0) = 4$

$\frac{d^2y}{dt^2} + y = t$ $y(0) = 1$, $y'(0) = 0$

$\frac{d^2y}{dt^2} + 9y = \cos 2t$ $y(0) = 1$, $y(\pi/2) = -1$

$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y = 2$ $y(0) = 1$, $y'(0) = 0$